CMPUT 307

Quiz 1, Upload by Jan. 21, 2022 (max marks 76)

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1. [40] Consider an 10x10 image with six possible gray levels, 0,1,2,3,4,5 as shown below:

0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 1 1 1 1 1

2 2 2 2 2 2 2 2 2 2

2 2 2 2 2 2 2 2 2 2

3 3 3 3 3 3 3 3 3 3

3 3 3 3 3 3 3 3 3 3

3 3 3 3 3 3 3 3 3 3

4 4 4 4 5 5 5 5 5 5

5 5 5 5 5 5 5 5 5 5

5 5 5 5 5 5 5 5 5 5

For parts (a) to (c) assume binary coding (i.e., only 0 & 1 transmitted).

1. [4] Calculate probabilities of the six gray levels.

Ans. Probability by level: 0(15%), 1(5%), 2(20%), 3(30%), 4(4%), 5(26%)

1. [4] What is the Entropy of this set of symbols?

Ans. Approximately 2.3032 to 5 s.f.

1. [12] Apply reduction process to derive the Huffman codes.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Symbol** | **Probabilities** | **Reduction 1** | **Reduction 2** | **Reduction 3** | **Reduction 4** |
| 4 | 0.04 |  |  |  |  |
| 1 | 0.05 | 0.09 |  |  |  |
| 0 | 0.15 | **0.15** | ***0.24*** |  |  |
| 2 | 0.20 | **0.20** | **0.20** | 0.44 | 0.44 |
| 5 | **0.26** | 0.26 | **0.26** | 0.26 | 0.56 |
| 3 | **0.30** | **0.30** | 0.30 | 0.30 |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Symbol** | **Code Step 5** | **Code Step 4** | **Code Step 3** | **Code Step 2** | **Code Step 1** |
| 4 | 1011 |  |  |  |  |
| 1 | 1010 | 101 |  |  |  |
| 0 | 100 | 100 | 10 |  |  |
| 2 | 11 | 11 | 11 | 1 | 1 |
| 5 | 01 | 01 | 01 | 01 | 0 |
| 3 | 00 | 00 | 00 | 00 |  |

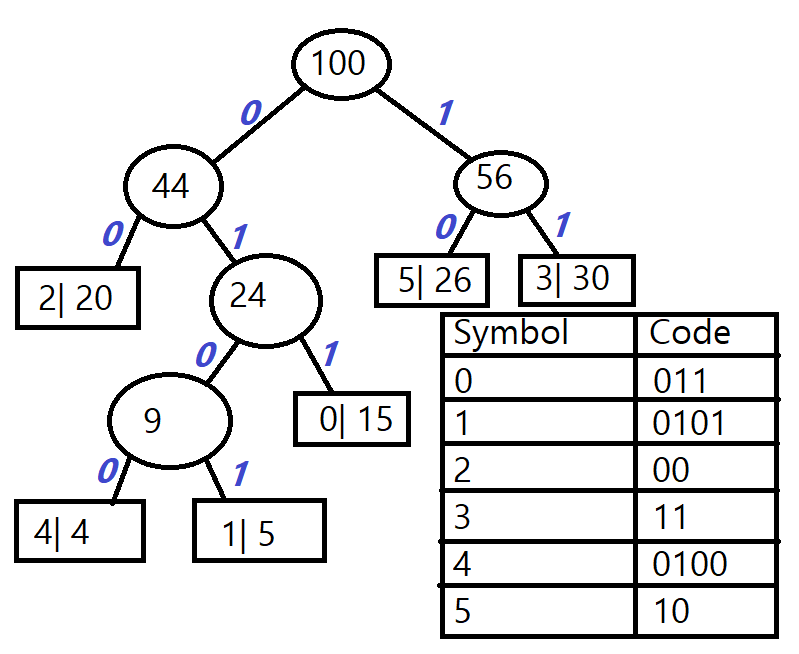
1. [4] What is the Average Code length for the Huffman Codes in Part (b)?

Ans. 2.33

1. [4] What is the Efficiency of the Huffman coding for this specific problem? [Where Efficiency is defined as Entropy/(Average Huffman Code Length).]

Ans. 2.3032 / 2.33 = 0.98850 (to 5 s.f.)

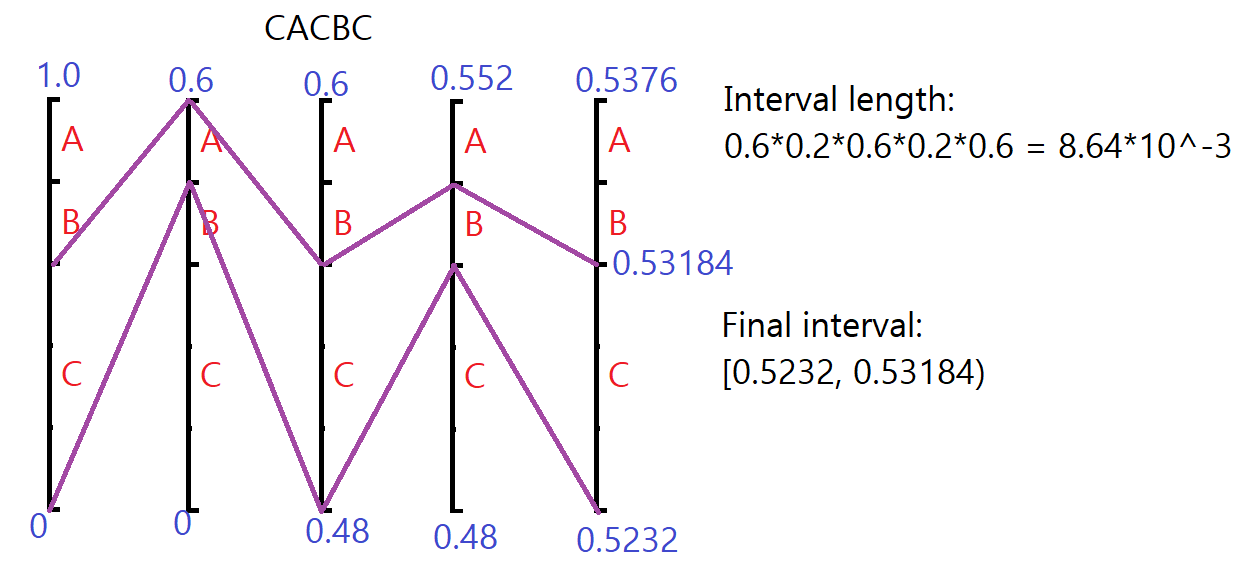
1. [12] Instead of using the symbol reduction process in Part (c), use the Huffman Tree to generate the codes below.



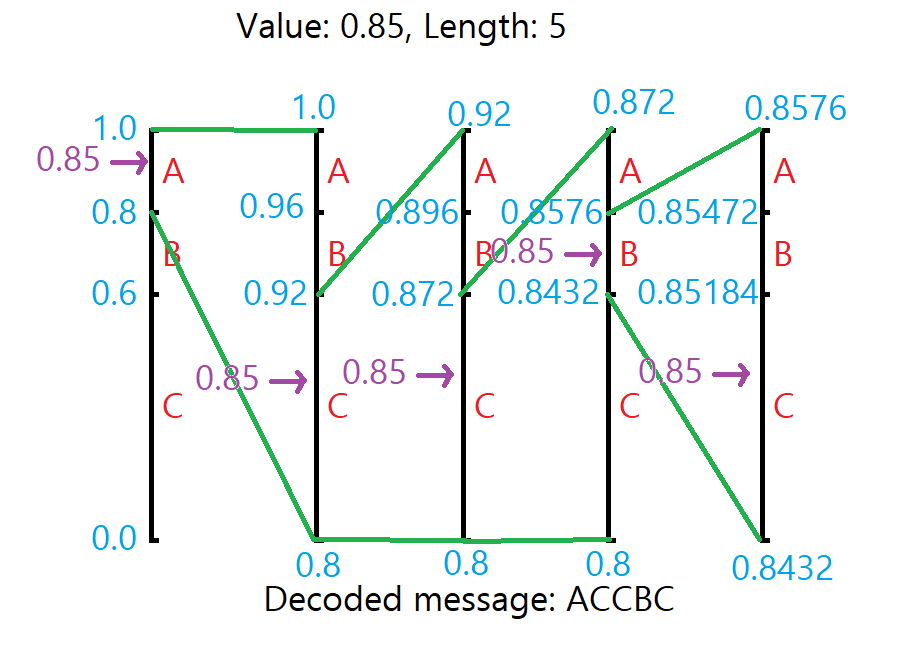
1. [36] **Arithmetic Coding**

Consider three symbols A, B, C and a string of 5 characters containing 3 Cs, 1 A, and 1 B coded using Arithmetic coding. Assume that the intervals for A, B, and C are arranged from top to bottom when the code is generated.

1. [10] Show below the steps involved in generating the message interval for CACBC.



1. [10] Show below the steps involved in decoding the value 0.85.



1. [6] What are the code intervals for the Arithmetic Codes given by “0010”, “1100” and “101”?

Ans. 0010 = [0.125, 0.15625)

1100 = [0.75, 0.78125)

101 = [0.625, 0.6875)

1. [10] Show below the steps involved in determining the Coding Interval from the Message Interval in Step (a). What is the Arithmetic Code for this Coding Interval?

Ans.

Convert 0.5232 and 0.53184 to binary = 0.10000101111100000111 and 0.10001000001001101011

Truncating the lower bound to 8 decimal places = 0.10000101, which is 0.5195, lower than 0.5232, so we increase it to 0.10000111, which is 0.5273, greater than 0.5232.

Then we add an additional 1 to the last binary number and make sure that it does not exceed 0.53184.  
0.100001111 ≈ 0.5293, so the binary number 0.10000111 gives a range within the upper and lower bounds. Hence the encoded message will be

10000111